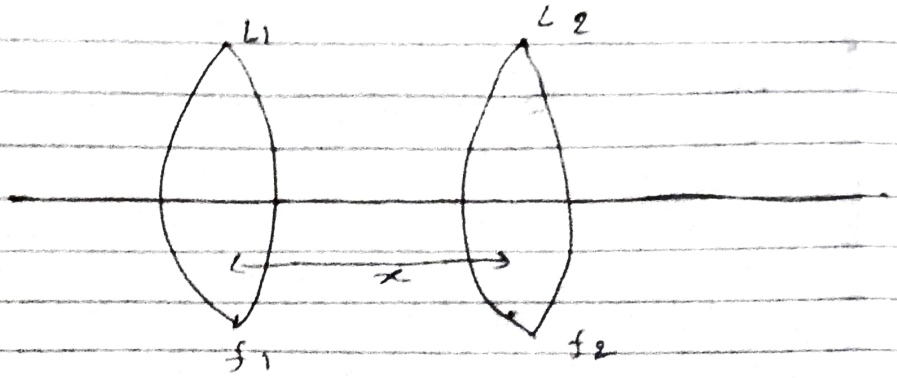
condition for achromatism of two thin lenses separated by a finite distance



$$x = \frac{f_1 + f_2}{2}$$

Two thin lenses of made of some material and separated by a distance is that the distance between two lenses must be equal to the mean focal length of two lenses. Then this combination free from chromatic aberration

Formula for equivalent focal length of two lenses separated by a x distance.

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{x}{f_1 f_2}$$

Differentiate above eqn w.r.t f

$$\frac{d}{df}\left(\frac{1}{f}\right) = \frac{d}{df}\left(\frac{1}{f_1} + \frac{1}{f_2} - \frac{x}{f_1 f_2}\right)$$

$$-\frac{1}{f^2} = -\frac{1}{f_1^2}\left(\frac{df_1}{df}\right) - \frac{1}{f_2^2}\left(\frac{df_2}{df}\right) -$$

$$\left[\frac{x}{f_1}\left(-\frac{1}{f_2^2}\frac{df_2}{df}\right) + \frac{x}{f_2}\left(-\frac{1}{f_1^2}\frac{df_1}{df}\right)\right.$$

$$\frac{-df}{f^2} = -\frac{df_1}{f_1^2} - \frac{df_2}{f_2^2} - \left[\frac{x}{f_1}\left(\frac{-df_2}{f_2^2}\right) + \frac{x}{f_2}\left(\frac{-df_1}{f_1^2}\right)\right]$$

$$-\frac{df}{f^2} = \frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} - \left[\frac{x}{f_1}\frac{\omega_2}{f_2} + \frac{x}{f_2}\frac{\omega_1}{f_1}\right]$$

$$-\frac{df}{f^2} = \frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} - \left(\frac{x\omega_2}{f_1 f_2} + \frac{x\omega_1}{f_1 f_2}\right)$$

For achromatism

$$df = 0$$

$$0 = \frac{\omega_1 f_2 + \omega_2 f_1}{f_1 \cdot f_2} - x\left(\frac{\omega_2 + \omega_1}{f_1 f_2}\right)$$

$$\frac{\omega_1 f_2 + \omega_2 f_1}{f_1 f_2} = x\,\frac{(\omega_2 + \omega_1)}{f_1 f_2}$$

$$x = \frac{\omega_1 f_2 + \omega_2 f_1}{\omega_1 + \omega_2}$$

$$\omega_1 = \omega_2 = \omega$$

$$x = \frac{\omega\,(f_1 + f_2)}{2\omega}$$

$$\boxed{x = \frac{f_1 + f_2}{2}}$$

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